

Course Code/Title:	PHYS402- Nuclear Physics II (3C.U)
Date:	7 th April, 2013
Time:	2:00 am to 3:30 pm
Instructions:	Attempt ALL Questions.
Time Allowed:	1 hr 30mins.

- (a)
- Distinguish between charged and uncharged particles. (2mrks)
 - Define range R of a particle. (2mrks)
 - Give three examples of charged particles that can directly ionize matter. (1mrk)
 - What is the range of 2 MeV alpha particle in water? For H_2O (11%H, 89%O)
Atomic weights for hydrogen and oxygen are 1 and 16 respectively. (5mrks)
 - Determine the range of 10 MeV alpha particle in aluminum given that density of aluminum = $2.7 \times 10^3 \text{ kgm}^{-3}$ (3mrks)
- (b)
- A broad beam of neutrons is normally incident on a slab 5cm thick. The intensity of neutrons transmitted through the slab without interaction is found to be 25% of the incid intensity
- What is the total interaction coefficient μ , for the slab materials? (3mrk)
 - What is the average distance a neutron travel in this material before undergoing an interaction? (3mrks)
- (a)
- Give the mechanisms by which charged particles traveling through matter loose energy. (4mrks)
 - Write down the expression of energy loss due to bremsstrahlung emission for electron or positron in terms of stopping power due to ionization-excitation. (3mrks)
 - Consider an electron with $T=5\text{MeV}$. What fraction of its energy is lost as bremsstrahlung as it stops moving in Aluminum ($Z=13$)? (3 mrks)
 - What is the total energy radiated in (ii) above? (2mrks)
- (b)
- Write down the expression of range of proton and deuteron (2mrk)
 - What is the range of 5Mev deuteron in air? (3mrks)
 - Define half-thickness and give its expression. (1mrk)

iii) Calculate the half-thickness for $\mu_t = 20\text{m}^{-1}$. (2mrks)

3.

- a)
- Write down the expression of the mass absorption coefficient for transmission of Beta particles. (2mark)
 - What is the value of the absorption coefficient for 2 MeV betas? (3marks)
 - What fraction of 2 MeV betas will go through a single Au foil of thickness 0.1mm? (3marks) $\rho_{Au} = 19.3 \times 10^3 \text{ kg / m}^3$

- b)
- Give a brief explanation of the photoelectric effect. Write down the expression of the photoelectric coefficient defining all the terms involved. (4 marks)
 - Calculate the energy of a light photon of wavelength $0.7\mu\text{m}$. (2marks)
 - When this photon strike a certain metal. Calculate the velocity of the exited electron whose binding energy is equal to 2.5eV . (2marks)
- c)
- What is Pair production? (2 mrks)
 - Write down the expression of the pair production cross section. (2 mrks)

GOMBESTATEUNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF PHYSICS
2018/2019 SESSION SECOND SEMESTER
PHYS402 TEST No1

INSTRUCTION: ANSWER ALL
TIME: 2 hours

1.
 - a) Differentiate between moving charge and uncharged particle.
 - b) Define ionizing power and write down its expression.
 - c.) What are the principal features that characterize the passage of charged particles in matter?
 - d) Write down the interactions responsible for the principal features in c.) above.
2.
 - a) Calculate the range in air for 10 MeV protons.
 - b) Calculate the range of 10 MeV Proton in water. Density of water is 1000 kg/m^3
 - c) What is the range of 3MeV deuteron in aluminum using the Braggs Kleeman rule and the result in 2 a) above.
3.
 - a) What are the light charged particles?
 - b) Write down the mechanisms through which light charged particles deposit energy and explain briefly each mechanism.
 - c) Write down the expression of energy loss due to bremsstrahlung emission and define all terms involved.
4. Consider an electron with $T=5\text{MeV}$. What fraction of its energy is lost as bremsstrahlung as it starts moving in:
 - a) Aluminum and
 - b) Lead ($Al_z = 13$ and $P_b = 82$)
 - c) Calculate the total energy radiated in 2 c) above
5.
 - a) Write down the expression of the total stopping power for electron or positron.
 - b) What is the expression of the effective atomic number Z_{ef} for a particle moving in a compound?
6. Use the expression in 5 b) to calculate Z_{ef} for the followings:
 - a) a particle moving in water.(water consists of 11% H and 89% O)
 - b) a particle moving in air. (Air consists of 22.9% O, 74.5% N and 2.6% Ar.
Take: $\{ m(O)=16; m(N)=14; m(Ar)=40; Z(O)=8; Z(N)=7; Z(Ar)=18 \}$

PHY 402, 7th April, 2013

1hr, 30mins.

1a) charged particles	uncharged particles
i) they interact directly with electrons and nuclei through Coulombic interactions.	they cannot interact directly with electrons and nuclei through Coulombic interactions.
ii) they are directly ionizing the medium	they are indirectly ionizing medium.
iii) they have low penetrating ability	iii) they have high penetrating ability
iv) e.g. electrons, protons, alpha particles, beta particles	neutrons, gamma rays, "n", "γ"

1a ii) Range "R" of a particle: the distance that a particle travels from its source through matter, and thus losing energy in many steps, until its energy is (almost) zero.

1a iii) ① proton "¹H", ② electron "⁰e"

③ Alpha particles "⁴α"

$$1a iv) * \frac{1}{\sqrt{A_{ef}}} = \left[\sum_{i=1}^L \frac{w_i}{\sqrt{A_{fi}}} \right]^{-1} = \left[\frac{w_H}{\sqrt{A_H}} + \frac{w_O}{\sqrt{A_O}} \right]^{-1}$$

$$\Rightarrow \left\{ \begin{aligned} w_H &= \frac{10\%}{100} = 0.11, w_O = \frac{89\%}{100} = 0.89 \end{aligned} \right\}$$

$$\sqrt{A_{ef}} = \left[\frac{0.11}{\sqrt{1}} + \frac{0.89}{\sqrt{16}} \right] = 3 \Rightarrow \boxed{\sqrt{A_{ef}} = 3} \quad (1)$$

$$\therefore (\sqrt{A_{ef}})^2 = 3^2 \quad \underline{A_{ef} = 9}$$

$$* R(mm) = \exp(1.61 \sqrt{T(MeV)}), 1 < T \leq 4 MeV$$

$$R(mm) = (0.05T + 2.85) T^{3/2}, 4 \leq T \leq 15 MeV$$

Since $T = 2 MeV$ we have;

$$R_{air}(mm) = \exp(1.61 \sqrt{2}) = 9.7462620832 mm$$

$$\boxed{R_{air} \approx 9.75 mm} \quad (2)$$

Using the relation

$$R(mm) = (3.2 \times 10^{-1}) \frac{\sqrt{A_{ef}}}{\rho(kg/m^3)} R_{air}(mm) \quad (3)$$

table $\rho_{water} = 1 \times 10^3 kg/m^3$

①

And by substituting eqn ①, ② into ③ we have;

$$R_{water}(mm) = (3.2 \times 10^{-1}) \left[\frac{3}{1 \times 10^3} \right] (9.75)$$

$$= 3.2 \times 10^{-1} (3 \times 10^{-3}) \times (9.75)$$

$$R_{water}(mm) = 9.36 \times 10^{-3} mm = 9.36 \times 10^{-3} \times 10^3$$

$$R_{water}(mm) = 9.36 \times 10^{-6} m = \underline{\underline{9.36 \mu m}}$$

1a v) "If" range in silicon is given we can use bragg-Kleem rule as; (e.g. $R_s = 70 \mu m$)

$$\Rightarrow R_{Al} = \frac{\rho_{Si}}{\rho_{Al}} \sqrt{\frac{A_{Al}}{A_{Si}}} R_{Si} = \frac{2.33 \times 10^3}{2.7 \times 10^3} \sqrt{\frac{27}{28}} \times 70 \mu m$$

$$R_{Al} = 59 \mu m$$

"But" if it's not given we can use bragg-Kleem rule as;

$$\Rightarrow R_{Al} = (3.2 \times 10^{-1}) \frac{\sqrt{27}}{2.7 \times 10^3} [0.05(10) + 2.85] (10)^{3/2} mm$$

$$R_{Al} = 6.52 \times 10^{-5} m = \underline{\underline{65 \mu m}}$$

NB: $65 - 59 = 6 \mu m = 0.000006 m$ is within the range of accuracy of bragg-Kleem rule.

$$1b i) I = I_0 e^{-\mu x} \quad \text{where:} \quad (4)$$

Data

$x = 5 cm$

$\mu = ?$

$$\frac{I}{I_0} = \frac{25\%}{100} = 0.25$$

"I" is intensity after attenuation

" I_0 " is incident intensity

" μ " linear attenuation (for good geometry) (cm^{-1})

"x" physical thickness of the absorber (cm)

we can rewrite eqn ④ as;

$$\frac{I}{I_0} = e^{-\mu x} \Rightarrow \ln\left(\frac{I}{I_0}\right) = \ln[e^{-\mu x}]$$

$$= -\mu x [\ln e] = -\mu x [1]$$

$$\therefore \mu = -\frac{\ln\left(\frac{I}{I_0}\right)}{x} = -\frac{\ln(0.25)}{5} = -\frac{(-1.386)}{5} = 0.277$$

$$\boxed{\mu = 0.277 cm^{-1}}$$

ii) $\bar{x} = \frac{1}{\mu} = \frac{1}{0.277} \approx 3.61 cm$ (this average travel distance between interaction, " $\frac{1}{\mu}$ " is called the "mean-free-path length".)

②

2a) There are two mechanisms in which light charged particle (i.e. electron β^- and positron β^+) that travel through matter lose energy this are:

(a) Collisional losses: electrons lose energy via interactions with orbital electrons in the medium

* This leads to excitation of the atom or ionization

* Maximum energy transfer occurs in a "head on" collision between two particles of mass "m" and "M" and can be expressed as;

$$Q_{max} = \frac{mME}{(M+m)^2}$$

(b) Radiative losses: bremsstrahlung.

* A charged particle undergoing a change in acceleration always emit "radiative" electromagnetic radiation called bremsstrahlung.

* The larger the change in acceleration, the more energetic the ~~change~~ bremsstrahlung photon.

$$2a(ii) \left[\frac{dE}{dx} \right]_{rad} = \frac{Z T (MeV)}{750} \left[\frac{dE}{dx} \right]_{ion}$$

where $\left[\frac{dE}{dx} \right]_{ion}$ is the stopping power due to ionization - excitation.

→ "Z" is the atomic weight of medium which electron/positron pass through.

→ "T (MeV)" is the kinetic energy of incident electron/positron.

$$2a(iii) T = 5 \text{ MeV}, \left[\frac{dE}{dx} \right]_{rad} = ?, Z = 13$$

$$\begin{aligned} \text{by using } \left[\frac{dE}{dx} \right]_{rad} &= \frac{Z T (MeV)}{750} \left[\frac{dE}{dx} \right]_{ion} \\ &= \frac{13 \times 5}{750} \left[\frac{dE}{dx} \right]_{ion} = \frac{65}{750} \left[\frac{dE}{dx} \right]_{ion} \\ \left[\frac{dE}{dx} \right]_{rad} &\approx 0.09 \left[\frac{dE}{dx} \right]_{ion} \end{aligned}$$

that is the rate of energy loss due to radiation is about "9%" of $\left[\frac{dE}{dx} \right]_{ion}$

$$\begin{aligned} 2a(iv) \text{ by using } T_{rad} &= 4.0 \times 10^{-4} Z T^2 \\ T_{rad} &= 4.0 \times 10^{-4} (13) \times 5^2 = \underline{0.130 \text{ MeV}} \end{aligned}$$

(3)

$$2b(i) R_{(pd)} = \left[\frac{4 M_{\alpha}}{m_{(pd)}} \right] R_{\alpha} - 2(m_{air})_{(1,2)}$$

where R_{α} = range of α particles in air having the same speed as deuteron or the proton.

→ $m_{(pd)}$ = mass of particles (1 for proton, 2 for deuteron)

→ M_{α} = mass of alpha particle = 4

2b(ii) For K.E of alpha particle with same speed of deuteron we have;

$$T_{\alpha} = \frac{1}{2} M_{\alpha} V_{\alpha}^2 \quad \text{--- (i)} \quad [V_{\alpha} = V_d]$$

$$T_d = \frac{1}{2} M_d V_d^2 \quad \text{--- (ii)}$$

by ~~equating~~ ^{dividing} eqn (i) & (ii) we can write;

$$\frac{T_{\alpha}}{T_d} = \frac{M_{\alpha}}{M_d} \Rightarrow T_{\alpha} = \left[\frac{M_{\alpha}}{M_d} \right] T_d$$

but $M_{\alpha} = 4$ & $M_d = 2$

$$\therefore T_{\alpha} = \left[\frac{4}{2} \right] T_d = 2 T_d = 2 [5] = \underline{10 \text{ MeV}}$$

And, the range of 10 MeV alpha particle in air is; (i.e. 10^6 at range of $4 \leq T \leq 15 \text{ MeV}$).

$$\begin{aligned} R_{\alpha} &= \{0.05T + 2.85\} T^{3/2} = \{0.05(10) + 2.85\} 10^{3/2} \\ &= [3.35] [31.62] = 105.927 \approx 106 \text{ mm} \\ R_{\alpha} &\approx 106 \text{ mm} = 0.106 \text{ m} \end{aligned}$$

The range of the 5 MeV deuteron in air is;

$$R_d = 2 R_{\alpha} - 2 = 2(106) - 2 = 212 - 2 = 210 \text{ mm}$$

$$R_d = \frac{210 \text{ mm}}{1000} = \underline{0.21 \text{ m}}$$

2b(iii) (a) Half-thickness: It is the thickness of an absorber at which the intensity of the incident beam falls to its half value.

$$\mu_t = \frac{0.693}{HVT} \quad \text{"Half value thickness" (HVT)}$$

$$2b(iv) \text{ or } HVT = \frac{0.693}{\mu_t} = \frac{0.693}{200 \text{ m}^{-1}} = \underline{0.03465 \text{ m}}$$

$$HVT = X_{1/2}$$

(4)

3ai) $\mu (m^2/kg) = 1.7 E_{max}^{1.14}$
 where E_{max} is maximum kinetic energy of beta particles (MeV)

3aii) $E_{max} = 2 \text{ MeV}$

$\mu = 1.7 (2)^{1.14} = (1.7)(2.204) \approx 3.75 m^2/kg$

$\mu_{betas} \approx 3.75 m^2/kg$

3aiii) $N(t) = N(0) e^{-\mu t} \Rightarrow \frac{N(t)}{N(0)} = e^{-\mu t} = ?$

given

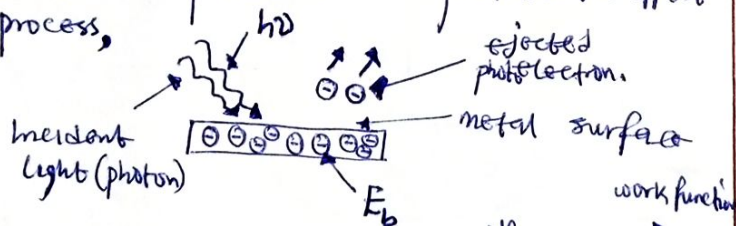
$E_{max} = 2 \text{ MeV}$ (same as above) $\Rightarrow \mu = 3.75 m^2/kg$

$t = 0.1 \text{ mm} = 0.1 \times 10^{-3} \text{ m}$ (thickness "t")

$\therefore \frac{N(t)}{N(0)} = e^{-[3.75][0.1 \times 10^{-3}]} = e^{-(3.75 \times 10^{-4})} = 0.9996$

$\frac{N(t)}{N(0)} \approx 1$

3bi)* In the photoelectric absorption process, a "photon" undergoes an interaction with an absorber "atom" in which the "photon" completely disappears. The process is called photoelectric effect process.



$E_e = h\nu - E_b$

$E_b = \phi = W$
 binding energy

where: " $h\nu$ " is the energy of the incident photon and " E_b " is the binding energy of the photoelectron in its original shell.

3bii) $E = h\nu$, but $\nu = \frac{c}{\lambda} \Rightarrow E = \frac{hc}{\lambda}$

$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{0.7 \times 10^{-6}} = 2.837 \times 10^{-19} \text{ J}$

$E \approx 1.77 \text{ eV}$

given

$\lambda = 0.7 \mu\text{m} = 0.7 \times 10^{-6} \text{ m}$

3biii) given

$E_b = 2.5 \text{ eV} = 4.01 \times 10^{-19} \text{ J}$

when the same photon (3bii) strikes a metal we have;

$E = h\nu = 1.77 \text{ eV} = 2.837 \times 10^{-19} \text{ J}$

thus;

$E_e = h\nu - E_b$

but $E_e = \frac{1}{2} m_e v_e^2$ & $v = \frac{c}{\lambda}$

$\therefore \frac{1}{2} m_e v_e^2 = \frac{hc}{\lambda} - E_b$

$m_e v_e^2 = (2.837 \times 10^{-19} - 4.01 \times 10^{-19}) \times 2$
 $= (-1.173 \times 10^{-19}) \times 2$

$m_e v_e^2 = -2.346 \times 10^{-19}$

$v_e^2 = \frac{-2.346 \times 10^{-19}}{9.11 \times 10^{-31}} = -2.58 \times 10^{11}$

$v_e = \sqrt{-2.58 \times 10^{11}} = \sqrt{-1} \times \sqrt{2.58 \times 10^{11}}$

$v_e = i 5.08 \times 10^5 \text{ ms}^{-1}$ $\boxed{\sqrt{-1} = i}$

This means that:

NO emission if photon energy < workfunction (or binding energy)

i.e. $h\nu < E_b$ no emission

emission occurs only when $h\nu > E_b$

3ci) pair production: occurs when the gamma ray energy exceeds twice the rest mass energy of an electron (1.02 MeV). By that effect the gamma-ray is transformed to matter in the form of a pair of negatively and positively charged electrons (negatron and positron).

3cii) The expression of the cross section of the high energy pair production " σ_{pp} " can be written as;

$\sigma_{pp} \propto Z^2 \ln E$

where: " E " is energy of the photon

" Z " is atomic number of the absorber.

TEST NO.1

2018/2019

2 hours

1a) same as previous one.

1b) Ionizing power: is the amount of ionization produced by a charged particle per unit path length.

$$I = \frac{m\beta^2}{K.E}$$

where I is the ionizing power m is the mass of the particle β is the number of unit charge it carries $K.E$ is the it's kinetic energy.

1c) (i) Loss of energy the particles

(ii) Deflection of the particle from it's incident direction.

1d) The interaction responsible for the principle features in (c) above, are;

i) Inelastic collision with orbital electrons (excitation and ionization of atoms).

ii) Radiative losses in the field of nuclei (Bremsstrahlung emission).

iii) Elastic scattering with nuclei.

(iv) Elastic scattering with orbital electrons.

$$2a) T_p = 10 \text{ MeV}, T_\alpha = \frac{M_\alpha}{M_p} T_p = \left[\frac{4}{1} \right] [10]$$

$$T_\alpha = 40 \text{ MeV}$$

the range of 40 MeV alpha particles in air is;

$$R_\alpha = [0.05T + 2.85] T^{3/2} \quad 3/2 = 1.5$$

$$= [0.05(40) + 2.85] [40]^{1.5}$$

$$= [2 + 2.85] [40]^{1.5} = [4.85] [253.0]$$

$$R_\alpha = 1227 \text{ mm}$$

$$\text{For proton; } R_p = \frac{4[M_\alpha]}{[M_p]} R_\alpha - 2 = 4 \left[\frac{1}{4} \right] R_\alpha - 2$$

$$= R_\alpha - 2 = 1227 - 2 = 1225 \text{ mm}$$

$$R_p = 1225 \text{ mm}$$

(7)

2b) Using the result in 2a) the range of 10 MeV proton is 1225 mm in air. $\therefore$ (i.e. $R_p = 1225 \text{ mm}$) and molecular mass of water (i.e. A_{ef}) is $[2 \times 1 + 16] = 18$
 $A_{ef} = 18, \rho_{H_2O} = 1000 \text{ kg/m}^3$
 Thus, $53.49 \mu\text{m}$

$$R_p = (3.2 \times 10^{-1}) \frac{\sqrt{A_{ef}}}{\rho (\text{kg/m}^3)} R_{air} (\text{mm})$$

$$= [3.2 \times 10^{-1}] \frac{\sqrt{18}}{1000} [1225]$$

$$= \frac{1663.12}{1000} = 1.663 \text{ mm} \Rightarrow R_p = 1663 \mu\text{m}$$

2b using the result in 2a) the range of 10 MeV proton is 1225 mm in air $\Rightarrow$ (i.e. $R_{p(air)} = 1225 \text{ mm}$) and A_{ef} of H_2O is $\Rightarrow \sqrt{A_{ef}} = \left(\frac{0.11}{\sqrt{1}} + \frac{0.89}{\sqrt{16}} \right)^{-1} = 3 \Rightarrow \boxed{\sqrt{A_{ef}} = 3}$

Thus

$$R_{p(H_2O)} = [3.2 \times 10^{-1}] \frac{\sqrt{A_{ef}}}{\rho (\text{kg/m}^3)} R_{air} (\text{mm})$$

$$= \frac{[3.2 \times 10^{-1}] [3] [1225]}{1000}$$

$$R_{p(H_2O)} = 1.176 \text{ mm} = 1176 \mu\text{m}$$

2c, the result in 2a) is $R_{p(air)} = 1225 \text{ mm}$

$$R_d(Al) = 3.2 \times 10^{-1} \times \frac{\sqrt{27}}{2.7 \times 10^{-3}} \times 1225 = 754.4$$

$$R_d(Al) = 754.4 \times 10^3 \text{ mm} = 754 \text{ m}$$

3) same as previous one

4a) same as previous one

4b) The same electron moving in lead $\left(\frac{dE}{dx} \right)_{rad} = \frac{82(5)}{750} \left(\frac{dE}{dx} \right)_{ion} = 0.55 \left(\frac{dE}{dx} \right)_{ion}$ In this case the rate of radiation energy is 55% of $\left(\frac{dE}{dx} \right)_{ion}$.

4c) same as previous one

(8)

5a) The total stopping power for electron or proton will be given by:

$$\left(\frac{dE}{dx}\right)_{\text{tot}} = \left(\frac{dE}{dx}\right)_{\text{ion}} + \left(\frac{dE}{dx}\right)_{\text{rad}} = \left[1 + \frac{ZT}{750}\right] \left[\frac{dE}{dx}\right]_{\text{ion}}$$

but $\left(\frac{dE}{dx}\right)_{\text{rad}} = \frac{ZT}{750} \left(\frac{dE}{dx}\right)_{\text{ion}}$

PROVED $\therefore \left(\frac{dE}{dx}\right)_{\text{tot}} = \left(\frac{dE}{dx}\right)_{\text{ion}} + \frac{ZT}{750} \left(\frac{dE}{dx}\right)_{\text{ion}} = \left[1 + \frac{ZT}{750}\right] \left[\frac{dE}{dx}\right]_{\text{ion}}$

where:

$\Rightarrow T(\text{MeV})$ is the K.E of electrons or positrons moving in a material

$\Rightarrow Z$ is the material atomic number

$\Rightarrow \left(\frac{dE}{dx}\right)_{\text{rad}}$ is the energy loss due to bremsstrahlung emission (radiative loss)

$\Rightarrow \left(\frac{dE}{dx}\right)_{\text{ion}}$ is the energy loss due to ionization.

5b) The value of Z_{ef} is given by

$$Z_{\text{ef}} = \frac{\sum_{i=1}^L \left[\frac{w_i}{A_i}\right] Z_i^2}{\sum_{i=1}^L \left[\frac{w_i}{A_i}\right] Z_i}$$

where "L" = number of element in compound

" w_i " = weight of fraction of i^{th} element

" A_i " = atomic weight of i^{th} element

" Z_i " = atomic number of i^{th} element

For a compound with molecular weight "M", the weight fraction is given by:

$$w_i = \frac{N_i A_i}{M}$$

where N_i is the number of atoms in the compound of i^{th} element.

$$\begin{aligned} 6a) Z_{\text{ef}} &= \frac{\left[\frac{w_H}{A_H} Z_H^2 + \frac{w_{\text{oxygen}}}{A_{\text{oxygen}}} Z_{\text{oxygen}}^2\right]}{\left[\frac{w_H}{A_H} Z_H + \frac{w_O}{A_O} Z_O\right]} \\ &= \frac{\left[\frac{0.11}{1} (1)^2 + \frac{0.89}{16} (8)^2\right]}{\left[\frac{0.11}{1} (1) + \frac{0.89}{16} (8)\right]} \\ &= \frac{[0.11 + 3.56]}{[0.11 + 0.445]} = \frac{[3.67]}{[0.555]} = 6.613 \end{aligned}$$

$Z_{\text{ef}} = 6.613$ For a particle moving in water (compound)

6b) For a particle moving in air (mixture)

$$\begin{aligned} Z_{\text{ef}} &= \frac{\left[\frac{w_{N_2}}{A_{N_2}} Z_{N_2}^2 + \frac{w_N}{A_N} Z_N^2 + \frac{w_{Ar}}{A_{Ar}} Z_{Ar}^2\right]}{\left[\frac{w_{N_2}}{A_{N_2}} Z_{N_2} + \frac{w_N}{A_N} Z_N + \frac{w_{Ar}}{A_{Ar}} Z_{Ar}\right]} \\ &= \frac{\left[\frac{0.229}{(16)} (8)^2 + \frac{0.745}{(14)} (7)^2 + \frac{0.026}{(40)} (18)^2\right]}{\left[\frac{0.229}{(16)} (8) + \frac{0.745}{(14)} (7) + \frac{0.026}{(40)} (18)\right]} \\ &= \frac{[0.916 + 2.6075 + 0.2106]}{[0.1145 + 0.3725 + 0.0117]} \\ &= \frac{[3.7341]}{[0.4987]} = 7.48766793664 \end{aligned}$$

$Z_{\text{ef(air)}} \approx 7.50$

"GOD SAVE US"